

BCB BCB/GDCB/STAT/COM S 568 Spring 2010

Homework 2

January 26, 2010

Due one week later. Answers to selected problems will be posted.

- 1) Represent a sequence over the alphabet R, Y as a Markov chain with states R and Y, with initial and transition probabilities all equal to 0.5. What is the expected waiting time to find the word "RYYR"? Please use two independent methods (first step analysis and renewal theory application) to calculate this probability and check your result.
- 2) Solve the problems posted in (1) for the word "RRRR" instead of "RYYR".
- 3) Under the same conditions as in (1) and (2), what is the probability that the word "RYRR" occurs before "YRYR"? How about "YRYR" versus "YRYR"?
- 4) Represent a sequence over the alphabet R, Y as a Markov chain with states R and Y, with initial and transition probabilities as follows:

$$\text{Initial probabilities: } \begin{cases} P(Y) = 0.4 \\ P(R) = 0.6 \end{cases}$$

$$\text{Transition probabilities: } \begin{cases} P(Y|Y) = 0.3 \\ P(R|Y) = 0.7 \\ P(Y|R) = 0.8 \\ P(R|R) = 0.2 \end{cases}$$

What is the expected waiting time to find the word "RYYR"?

- 5) Represent a sequence over the alphabet R, Y as a Markov chain with states R and Y, with initial and transition probabilities all equal to 0.5. What is the expected waiting time to find the pattern "RYYR" twice? Calculate under two different conditions: 1) The two occurrences are counted even if they overlap; 2) The two occurrences are required to be non-overlapping. Compare the results with your answer to (1).